Problem 32.21
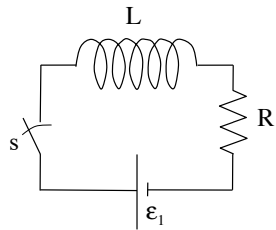
We know that: $R = 4\ \Omega$, $L = 1$ H, $\varepsilon = 10$ V

a.) The switch is closed at $t = 0$. As a function of time, what is the current in the inductor?
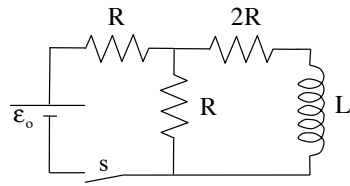
The inclination is to think that this is an easy problem. After all, we have an expression for current through an inductor. It is:

$$i(t) = \frac{\varepsilon_1}{R}\left(1 - e^{-t/(L/R)}\right)$$

The problem is that this expression is for a circuit that has a single resistor, a single power source and a single inductor. The circuit, in other words, shown to the right.

1.)

To get our circuit into that form, we would need to know the effective resistance in the circuit (not something that is easily done as none of the resistors are in either a parallel or series configuration), and essentially the voltage between points M and N as shown in the sketch (that would equal the voltage I've defined in the single element circuit as ε_1).
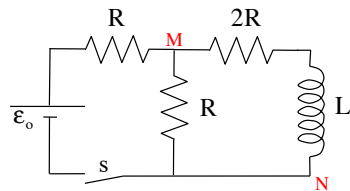
About our only chance is to use Kirchoff's Equations, try to generate an expression for the current through the inductor, and hope the final differential equation has the form:

$$\varepsilon_1 - R_{net}i_2 - L_1\frac{di_2}{dt} = 0$$

as this was the differential equation from whence came the solution:

$$i(t) = \frac{\varepsilon_1}{R_{net}}\left(1 - e^{-t/(L_1/R_{net})}\right)$$

2.)

All Kirchoff's Law problems start with a defining of current, then a writing of Node Equations (if not used in defining the currents, which is the way I'm going), then a writing of Loop Equations. With the currents defined in the sketch, the Loop Equations follow:
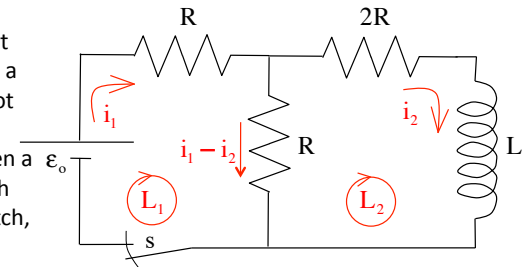
for L_1:

$$\varepsilon_o - (R)i_1 - (R)(i_1 - i_2) = 0$$

$$\Rightarrow\quad (2R)i_1 - (R)i_2 = \varepsilon_o \qquad \text{Equation A}$$

for L_2:

$$+(R)(i_1 - i_2) - (2R)i_2 - L\frac{di}{dt} = 0$$

$$\Rightarrow\quad (R)i_1 - (3R)i_2 = L\frac{di}{dt} \qquad \text{Equation B}$$

3.)

Equation A becomes:

$$(2R)i_1 - (R)i_2 = \varepsilon_o$$

$$\Rightarrow\quad i_1 = \frac{\varepsilon_o}{2R} + \frac{i_2}{2} \qquad \text{Equation C}$$

Substituting Equation C into Equation B and manipulating becomes:

$$(R)i_1 - (3R)i_2 = L\frac{di}{dt} \qquad \text{(substitute in } i_1 \text{ from Equ A)}$$

$$\Rightarrow\quad (R)\left(\frac{\varepsilon_o}{2R} + \frac{i_2}{2}\right) - (3R)i_2 = L\frac{di}{dt} \qquad \text{(redistribute)}$$

$$\Rightarrow\quad -\left(\frac{5}{2}R\right)i_2 + \frac{\varepsilon_o}{2} = L\frac{di}{dt}$$

$$\Rightarrow\quad +\frac{\varepsilon_o}{2} - \frac{5}{2}Ri_2 - L\frac{di}{dt} = 0$$

4.)

This equation suggests a circuit in which there is:

--a single resistor of resistance $R_{net} = \dfrac{5}{2}R = \dfrac{5}{2}(4 \ \Omega) = 10 \ \Omega$

--a single inductor of inductance $L_1 = 1 \ H$

--a single power supply $\varepsilon_1 = \dfrac{\varepsilon_o}{2} = 5 \ V$

Putting all of this information into our expression for the current through an inductor in this setting, we get:

$$i_{thru \ inductor} = \frac{\varepsilon_1}{R_{net}}\left(1 - e^{-t/(L_1/R_{net})}\right)$$

$$= \frac{(5 \ V)}{(10 \ \Omega)}\left(1 - e^{-t/((1 \ H)/(10 \ \Omega))}\right)$$

$$= .5\left(1 - e^{-10t}\right)$$

As I said, not a trivial problem.

5.)

b.) The current drawn from the battery is:

Taking our original Equation A, dividing by "R" and substituting in for i_2, we can write:

$$(2R)i_1 - (R)i_2 = \varepsilon_o$$

$$\Rightarrow \quad i_1 = \frac{1}{2}\left(\frac{\varepsilon_o}{R} + i_2\right)$$

$$\Rightarrow \quad i_1 = \frac{1}{2}\left(\frac{(10 \ V)}{(4 \ \Omega)} + \left(.5\left(1 - e^{-10t}\right)\right)\right)$$

$$= 1.25 + .25\left(1 - e^{-10t}\right)$$

6.)